Journal of Banking and Finance 14 (1990) 41-53. North-Holland

# THE EFFECTS OF DOMESTIC AND FOREIGN YIELD CURVES ON THE VALUE OF CURRENCY AMERICAN CALL OPTIONS

## Jongmoo Jay CHOI

Temple University, Philadelphia, PA. 19122, USA

### Shmuel HAUSER

Israeli Securities Authority and Hebrew University, Jerusalem, Israel

### Received September 1988, final version received April 1989

This paper examines the sensitivity of the values of foreign currency American call options to the domestic and foreign term structures of interest rates. Pricing performances of currency option models are compared with and without the term structure effects. It is shown that there exist significant pricing biases if flat yield curves are assumed, and that different shapes of domestic and foreign yield curves can have major impacts on currency option prices.

#### 1. Introduction

The uncertainty of interest rates has become a matter of major concern to both the academic and investment communities when they attempt to price derivative assets such as options. For currency options, the problem is more complicated because (1) there are not one but two – domestic and foreign – stochastic interest rates to worry about, and (2) the international interest rate differential may dictate the rationality and timing of exercising options. Underlying the interest rate uncertainty and expectation is the term structure of interest rates, or yield curves.

In a recent study on gold futures options, Bailey (1987) demonstrates that option models which assume flat yield curves may misprice options if yields fluctuate significantly or if the underlying asset price is correlated with interest rates. Adams and Wyatt (1987) report pricing biases in European call options when interest rate uncertainty is not acknowledged in the model. No attempt, however, has been made to investigate the effects of non-flat domestic and foreign yield curves on the value of foreign currency American call options. Numerous empirical studies on currency calls, testing both European and American option pricing models, employ deterministic interest rates and assume flat yield curves. In these studies, interest rates can change over time as data, but mispricing can still occur due to the disparity between the term structure of interest rates and the maturity of options. This paper examines the sensitivity of the values of foreign currency American call options to the domestic and foreign term structures of interest rates. The paper uses Garman-Kohlhagen (1983) and Grabbe (1983) models of European currency options modified to American options by the Barone-Adesi and Whaley (1987) method of estimating the early exercise premium. The Garman-Kohlhagen model introduces the foreign interest rates, and the Grabbe model incorporates stochastic interest rate changes, while the Barone-Adesi and Whaley method translates the closed-form European solution to an American one. We use the term structure model developed by Vasicek (1977) and others to analyze the sensitivity of currency call option prices to the different shapes of domestic and foreign yield curves, and to show how incorporation of bond yield structures matched with the option maturities improves the performance of currency call pricing models.

The models with and without the term structure effects are analyzed, simulated, and estimated. The results indicate that (a) the model which utilizes such yield structures performs markedly better than the model which assumes constant interest rates, (b) prediction errors are reduced when sloped yield curves are considered, and (c) estimates of volatility are sensitive to time and the shapes of domestic and foreign yield curves.

### 2. The models of foreign currency options

Building on the classic model of Black and Scholes (1973) regarding European options on domestically-traded underlying securities, any model of foreign currency options must deal with three issues: (a) incorporation of foreign as well as domestic interest rates into the model, (b) recognition of stochastic nature of interest rate changes, and (c) consideration of the carly exercise premium of American options.

The first issue arises from the fact that default risk-free foreign bonds, as well as domestic bonds, represent a risk-free alternative to a hedged portfolio of spots and options on foreign exchange. Garman and Kohlhagen (1983) incorporate both foreign and domestic interest rates to obtain the following model of European currency call options:

$$c = e^{-r^*\tau} SN(d_1) - e^{-r\tau} XN(d_2), \quad \text{where}$$
<sup>(1)</sup>

 $\begin{aligned} &d_1 = d_2 + \sigma \sqrt{\tau} \\ &d_2 = [\ln{(S/X)} + (r - r^* - (\sigma^2/2))\tau]/\sigma \sqrt{\tau} \quad \text{and} \end{aligned}$ 

- c = the value of a European currency call option;
- S = the spot price of a unit of foreign currency in terms of domestic currency;
- X = the exercise price of the option;

- r = interest rate on default risk-free domestic pure discount bonds;
- $r^*$  = interest rate on default risk-free foreign pure discount bonds;
- $\tau$  = time to maturity of the option;
- $\sigma$  = standard deviation of the state variable(s) of the model; and
- N =cumulative normal distribution.

This model is based on the following dynamic diffusion process for S:

$$\frac{dS}{S} = \mu_s dt + \sigma_s dZ_s \tag{2}$$

and the partial differential equation (which results from imposing riskfree arbitrage):

$$0.5\sigma^2 S^2 \frac{\partial^2 c}{\partial S^2} - rc + (rS - r^*S) \frac{\partial c}{\partial S} - \frac{\partial c}{\partial \tau} = 0$$
(3)

subject to the boundary condition:

$$c(S,\tau) = \max\left[0, S - X\right],$$

where  $\mu_s$  is instantaneous expected value of S,  $\sigma_s$  is instantaneous standard deviation of S, and dz is the standard Wiener process. This model takes a single stochastic state variable S, and, therefore, assumes constant interest rates.

Grabbe (1983) considers the case of stochastic interest rates as reflected in the stochastic prices of pure discount bonds. In addition to (2), he specifies the following diffusion processes for the prices of domestic pure discount bonds B and foreign pure discount bonds  $B^*$ , denominated in their respective currencies:

$$\frac{dB}{B} = \mu_B dt + \sigma_B dZ_B \tag{4}$$

$$\frac{dB^*}{B^*} = \mu_{B^*} dt + \sigma_{B^*} dZ_{B^*}.$$
(5)

Price changes of foreign bonds in domestic currency unit can be stated as:

$$\frac{dG}{G} \equiv \frac{d(SB^*)}{SB^*} = (\mu_s + \mu_{B^*} + \rho_{sB^*}\sigma_s\sigma_{B^*})dt + \sigma_s dZ_s + \sigma_{B^*}dZ_{B^*}$$
$$\equiv \mu_G dt + \sigma_G dZ_G. \tag{6}$$

The following partial differential equation applies to both European and American options:

$$0.5\sigma_G^2 G^2 \frac{\partial^2 c}{\partial G^2} + \frac{\partial^2 c}{\partial G \partial B} G B \rho_{GB} \sigma_G \sigma_B + 0.5\sigma_B^2 B^2 \frac{\partial^2 c}{\partial B^2} - \frac{\partial c}{\partial \tau} = 0.$$
(7)

Unfortunately, the value of an American option is difficult to obtain analytically. A numerical method or an approximation must be used. Grabbe obtains the following analytic solution for European currency calls:

$$c = SB^*N(d_1) - XBN(d_2), \text{ where}$$

$$d_2 = \left[\ln\left(SB^*/XB\right) - (\sigma^2/2)\tau\right]/\sigma\sqrt{\tau}.$$
(8)

Since bond prices are related to interest rates via

$$B = e^{-rt}$$

$$B^* = e^{-r^*t},$$
(9)

the two models – Garman and Kohlhagen, and Grabbe – are identical in form, but they differ in their capacity to accommodate stochastic interest rate changes. Given the uncertainties in bond interest rates, the variance of the Grabbe model reflects the covariances of spot exchange rates and prices of domestic and foreign bonds:

$$\sigma^{2} = \int_{0}^{\tau} \frac{1}{\tau} (\sigma_{B}^{2} + \sigma_{G}^{2} - 2\sigma_{GB}) \,\mathrm{d}u.$$
(10)

In Garman and Kohlhagen, in contrast, bond rates are deterministic, and the model's volatility reduces to:

$$\sigma^2 = \int_0^t \frac{1}{\tau} \sigma_s^2 \,\mathrm{d}u. \tag{11}$$

For empirical evaluation, these models need to be restated for American options. Unlike a European option, an American option can be exercised prior to expiration. This extra flexibility implies that the value of an American option,  $C(S, \tau)$ , can be determined as the value of a European option,  $c(S, \tau)$ , plus the early exercise premium,  $\varepsilon(S, \tau) \ge 0$ :

$$C(S,\tau) = c(S,\tau) + \varepsilon(S,\tau).$$
<sup>(12)</sup>

Using a quadratic approximation, Barone-Adesi and Whaley (1987) developed a method of estimating the early exercise premium and of determining when the early exercise is optimal. Their method utilizes the fact that, since the same partial differential equation applies to both European and American options, it also applies to the early exercise premium. The quadratic approximation is thus applied to the early exercise premium, and the value of an American call option is

$$C = c + A_2 (S/S^c)^{q^2} \quad \text{when } S < S^c, \quad \text{and}$$

$$C = S - X$$
 when  $S \ge S^c$ ,

where  $S^c$  is the critical spot price of foreign currency. For determination of  $S^c$  as well as the definition of parameters, see their work (1987, pp. 306–307). In this paper, we estimate the early exercise premium according to the Barone-Adesi and Whaley method, and add it to the value of European options to obtain the value of American options.

Hsieh (1986) follows the same approach by combining the early exercise premium and European options. He examines the pricing of American options on Deutsche mark futures contracts, and finds the maturity bias of the Black and Scholes model. However, he does not trace the pricing bias to yield curves. The present paper, in contrast, investigates how the different shapes of domestic and foreign yield curves affect the prices of American options on the spot U.S.S prices of five major currencies.

Another method of determining the value of American currency options with stochastic interest rates is suggested by Ramaswamy and Sundaresan (1985). They use the numerical method to develop a model of American options on futures contracts, in which domestic interest rates are nonconstant and mean-reverting. Their model allows for two stochastic state variables - domestic interest rates and underlying commodity prices. To apply their model to the present empirical work, we would need to treat the domestic-foreign interest rate differential as one state variable. Although their model has an advantage of offering a one-step solution for American options, this treatment regarding the interest rate differential is not entirely satisfactory. In addition, implementation of their model requires assumptions on various speed of adjustment parameters and long-term stationary interest rates, as well as short-term expected values and variances. For these reasons, we choose to rely on a combination of the Grabbe model and the early exercise premium estimated by Barone-Adesi and Whaley, rather than using the Ramaswamy and Sundaresan model. As seen in (4)-(8), the Grabbe model does permit three state variables: spot exchange rates, and prices of domestic and foreign bonds. The present empirical work focuses on the effects of domestic and foreign yield curves, rather than those of random

interest rate fluctuations. The analysis of the effects of random interest rates per se is the subject of another paper.<sup>1</sup>

# 3. Empirical evaluation of currency option pricing models with or without the term-structure effects

In order to evaluate the effects of the term structures of interest rates we compare the performance of eq. (1) with flat yield curves (Model 1) against eq. (8) with sloped yield curves (Model 2) subject to adaptation to American options as indicated in eq. (12). To that end we first simulate the pricing bias of Model 1 relative to Model 2, and then we compare the tracking performance of the two models against historical data.

## 3.1. Sensitivity analysis

The effects of term structure on call values can be determined by examining the effects of time on bond prices in (9) and using  $\partial C/\partial B < 0$  and  $\partial C/\partial B^* > 0$ . In a world of stochastic interest rates, bond prices depend not only on the direct impact of time but on the indirect dependence of interest rates on time. From (9), we get:

$$\frac{\partial B}{\partial \tau} = -[r + \tau dr/d\tau]e^{-r\tau}$$

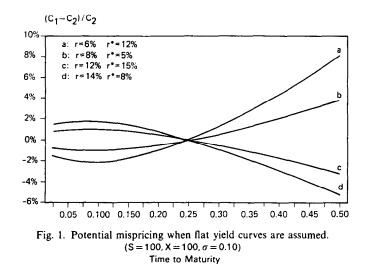
$$\frac{\partial B^{*}}{\partial \tau} = -[r^{*} + \tau dr^{*}/d\tau]e^{-r^{*}\tau}.$$
(13)

The pricing bias of the deterministic interest-rate models, thus, depends on the signs of  $dr/d\tau$  and  $dr^*/d\tau$ , i.e., upward or downward term structure of interest rates.

In this paper we focus on the effects of non-flat yield curves on option prices. By allowing interest rates to change each day, an aspect of interest rate uncertainty is indirectly captured here. But we do not directly deal with randomness in interest rates.

To further clarify why a model which permits sloped yield curves (Model 2) may better explain the observed behavior of currency call option market values, we illustrate the potential mispricing with the following simulation. The theoretical call values of Model 1 with flat yield curves,  $C_1$ , are calculated for all option maturities with the assumed parameter values of  $\sigma = 0.10$ , X = 100 and S = 100, along with 90 day domestic and foreign interest rates as indicated in fig. 1. The theoretical call values of Model 2,  $C_2$ , are then calculated for all maturities, and with the same parameter values. The domestic and foreign yields in Model 2, however, are yields with the

<sup>&</sup>lt;sup>1</sup>Ramaswamy and Sundaresan (1985, p. 1339) recognize the importance of incorporating a deterministic term structure, as well as consideration of random interest rate fluctuation in the model.



same maturities as those of call options. To generate such yields, we employ the following equations due to Vasicek (1977) and others.

$$y(r,\tau) = \pi + (r-\pi)(1-e^{-m\tau})/m\tau$$

$$y^{*}(r^{*},\tau) = \pi^{*} + (r^{*}-\pi^{*})(1-e^{-m^{*}\tau})/m^{*}\tau,$$
(14)

where  $\pi$  and  $\pi^*$ , respectively, are equilibrium values of domestic and foreign long-term interest rates, and *m* and *m*<sup>\*</sup> denote the speeds of adjustment towards them. For simulation purposes, it is assumed that  $\pi = \pi^* = 0.1$  and  $m = m^* = 1$  along with pairs of three-month interest rates for *r* and *r*<sup>\*</sup> as indicated in fig. 1.

By taking the first derivative with respect to time, the reader can verify that if  $r > \pi$  the domestic yield curve is a decreasing function of time to maturity, and an increasing function if  $r < \pi$ . Similar statements apply to foreign yield curves.

Summarizing different combinations of domestic and foreign sloped yield curves, all values are generated with Model 2,  $C_2$ , for four different cases:

- (a) an increasing domestic yield curve and a decreasing foreign yield curve;
- (b) an increasing domestic yield curve and an increasing foreign yield curve;
- (c) a decreasing domestic yield curve and a decreasing foreign yield curve; and

(d) a decreasing domestic yield curve and an increasing foreign yield curve.

These values are compared with values from Model 1 to compute the Model 1's bias relative to Model 2 as  $(C_1 - C_2)/C_2$ .

In fig. 1, with an upward-sloping domestic yield curve [cases (a) and (b)], the bias is negative for short-term options (less than three months), but for longer-term options the bias becomes positive. If the maturity of the option is three months, the bias is zero. Of course, there is nothing sacrosanct about the three-month period here; it simply reflects the fact that the three-month rate has been used as the bond interest rate in the simulation. Hence, if the time to maturity of the option is less than the maturity of the bond, the model underpredicts the actual market values; when the option maturity is longer than the bond maturity, the model overpredicts the market; when the two maturities are the same, the bias is zero. The absolute magnitude of the bias, however, is smaller in the case of (b) than (a) since the effects of domestic and foreign yield curves cancel out each other in the former cases but not in the latter. Similar points can be made for cases of (c) and (d). These simulations show that, depending on actual time remaining to expiration date, a foreign currency option model with single deterministic domestic and foreign interest rates (Model 1) can miss the market by a wide margin. Due to averaging, however, such biases may not be easily detectable in regression-type empirical work.

### 3.2. Empirical tests

To verify the relative empirical performances of the two models, we perform three experiments. First, we compute the implied standard deviations and the errors of the two models by currency, option maturity, and boundary status (in, at or out-of-the-money options). Second, we compare the tracking performance of the two models against historical data. Third, we regress the errors on several independent variables to see whether the errors contain significant systematic components.

The data used are synchronous transactions data of currency call options supplied by the Philadelphia Stock Exchange for the period of August 1984 to December 1985. There are 10,708 observations covering five currencies: the British Pound (BP), the Canadian Dollar (CD), the West German Mark (DM), the Japanese Yen (JY), and the Swiss Franc (SF). The risk-free bond yields are proxied by the annualized Eurocurrency deposit rates with maturities of 7, 30, 90, 180, and 360 days obtained from the *Financial Times*. Interest rates of different maturities are obtained by geometric interpolation based on the two flanking rates.<sup>2</sup>

<sup>&</sup>lt;sup>2</sup>The geometric interpolation is performed by calculating a daily compounding rate between flanking values and applying the rate to the beginning value for the time span concerned.

		Model 1		Model 2		MSE <sub>1</sub> /
	n	Av. ISD	Av. errors	Av. WTISD	Av. errors	MSE <sub>2</sub>
By currency						
BP	1,968	0.1131 (0.0316)	-0.0330 (0.7425)	0.1162 (0.0213)	-0.0531 (0.2544)	2.63
CD	715	0.0381 (0.0219)	-0.0947 (0.7013)	0.0403 (0.0134)	-0.1056 (0.3372)	3.54
DM	2,989	0.1105 (0.0237)	-0.2902 (0.3440)	0.1357 (0.0213)	-0.0835 (0.2519)	4.65
JY	2,298	0.1045 (0.0122)	-0.0524 (0.2781)	0.1124 (0.0111)	0.0047 (0.2165)	1.79
SF	2,738	0.1057 (0.0181)	0.1198 (0.4096)	0.1199 (0.0236)	0.0007 (0.2845)	1.54
Total	10,708					
	By maturity T < 30 $30 \le T \le 90$		n 1,105 3,893	MSE <sub>1</sub> / 1.31 2.94	MSE <sub>2</sub>	
		90 <i>&lt; T</i> Total	5,710 10,708	2.81		
	B	y boundary status				
	in-the-money at-the-money out-of-the-mone		1,890 5,149 7 3,669	2.54 2.87 2.76		
		Total	10,708			

Table 1 Implied standard deviations and errors of the two models.<sup>4</sup>

\*Numbers in the parentheses are standard errors. The weighted average of MSE ratios cannot be calculated based on the information given in this table because  $MSE_1/MSE_2 = \sum w_i MSE_1 / \sum w_i^* MSE_{2i}$  and  $w_i \neq w_i^*$  and  $MSE_{1i} \neq MSE_{2i}$ .

The purpose of the first test is to suggest that option's volatility is sensitive to time to maturity and the shape of domestic and foreign yield curves. Table 1 presents the implied standard deviations (ISD), average errors, and the ratios of mean squared errors (MSE) of the two models. The ISDs are calculated daily from the American option model without excluding any data. For model 1, the ISDs are computed using Whaley's (1982) procedure which minimizes the residual sum of squares between the model and market prices. For Model 2, in view of the importance of time, more weights are given to the options which are more sensitive to maturity effects by calculating the Weighted-Time Implied Standard Deviations (WTISD) rather than the ordinary ISDs:

$$WTISD_j = \sum_{j=1}^{n} w_{jt} ISD_{jt}, \text{ where}$$
 (15)

$$w_{jt} = (\partial C/\partial \tau)_{jt} / \sum_{j}^{n} (\partial C/\partial \tau)_{jt}.$$

The results reported in table 1 indicate that, with the weighted time adjustment, Model 2 has marginally higher ISDs. However, the ratio of MSEs unmistakably establishes that Model 2 is better. The MSEs are smaller, roughly by the order of two to one, in Model 2 than in Model 1 across all currencies, all option maturities and all boundary categories.

To compare the predictive power of the two models, the following linear regression is run:

$$C_{\rm mkt} = \hat{a}_0 + \hat{a}_1 C_{\rm mod} + \varepsilon, \tag{16}$$

where  $C_{mkt}$  is the actual market values of currency calls and  $C_{mod}$  is the theoretical values from Models 1 and 2. (The model values are calculated on day t based on the volatility estimates on day t-1.) It is posited that  $a_0=0$  and  $a_1=1$ . Errors are then decomposed using the Theil's method. Theil (1966) has shown that the MSEs can be decomposed as:

$$MSE = \frac{1}{n} \sum (C_{\text{mod}} - C_{\text{mkt}})^2$$
$$= (\bar{C}_{\text{mod}} - \bar{C}_{\text{mkt}})^2 + (1 - \hat{a}_1)^2 \sigma_{\text{mod}}^2 + (1 - \rho^2) \sigma_{\text{mkt}}^2,$$

where  $\rho$  is the correlation coefficient between  $C_{mod}$  and  $C_{mkt}$ , and the bar indicates the expected values. Division of both sides by MSE yields:

$$1 = U_1 + U_2 + U_3$$

where  $U_1$  is the bias proportion due to the systematic pricing errors of the model compared with actual values,  $U_2$  is the regression proportion due to the deviation of the regression slope from unity, and  $U_3$  is the disturbance proportion due to random fluctuations. With a perfect model, all the errors would be attributed to  $U_3$ .

Estimation results in table 2 indicate that Model 2 has higher  $R^2$ , smaller  $a_0$  (and with lower t values), and the value of  $a_1$  which is closer to unity (and with higher t values) than Model 1. Model 2 also has greater values of  $U_3$  than Model 1, indicating that the greater proportion of the Model 2 errors are random or that systematic mispricing is reduced in Model 2 compared to Model 1. Both of these results, again, confirm the superiority of Model 2 which incorporates non-flat yield curves.

The superior performance of Model 2 reflects the fact that, when employing Model 2, the maturities of the option and interest rate instruments are

	a <sub>0</sub>	<i>a</i> <sub>1</sub>	R <sup>2</sup>	$U_1$	$U_2$	U,
BP						
Model 1	0.115	0.986	0.917	0.037	0.013	0.950
	(7.15)	(2.08)				
Model 2	0.05	1.011	0.969	0.032	0.004	0.964
	(3.91)	(2.68)				
CD						
Model 1	0.163	0.797	0.796	0.063	0.125	0.812
	(12.2)	(1.29)				
Model 2	0.0623	0.972	0.911	0.123	0.007	0.870
	(7.92)	(2.39)				
DM						
Model 1	0.145	0.983	0.953	0.300	0.005	0.695
	(25.5)	(3.77)				
Model 2	0.014	1.002	0.985	0.021	0.001	0.978
	(4.13)	(4.00)				
JY						
Model 1	0.054	0.988	0.966	0.059	0.004	0.940
	(9.65)	(3.07)				
Model 2	0.0075	0.993	0.980	0.004	0.010	0.986
	(1.71)	(4.94)	0.700	0.001	0.010	01701
SF						
Model 1	0.0536	0.968	0.955	0.021	0.022	0.957
	(10.2)	(7.32)				
Model 2	-0.0036	0.968	0.968	0.007	0.030	0.963
MODEL 2	(-0.79)	(8.87)	0.700	0.007	0.050	0.902

 Table 2

 Tracking performances and the decomposition of errors.<sup>a</sup>

\*Numbers in the parentheses are t values.

aligned. (Model 1 employs interest rates that change daily, but not across option maturities.) Another interpretation is that the MSE of Model 2 is lower because more information is employed, namely, the maturities of various interest rate instruments. Thus the lower MSE is simply due to market efficiency; the pricing bias is smaller in Model 2 because investors are utilizing available information that is ignored when employing Model 1.

Finally, following Whaley (1982) we ran four sets of regressions:

$$(C_{\text{mod}} - C_{\text{mkt}})/C_{\text{mkt}} = \hat{\alpha}_0 + \hat{\alpha}_1 Q_i + \varepsilon, \qquad Q_i = S/X, \tau, \sigma, TS.$$
(17)

TS is the term structure variable calculated as:

$$TS = (r_{360} - r_7) - (r_{360}^* - r_7^*) = (r_{360} - r_{360}^*) - (r_7 - r_7^*)$$
 in Model 2, and

 $TS = r_{90} - r_{90}^*$  in Model 1, where subscripts indicate bond maturities.

	Model 1			Model 2		
	αο	α <sub>i</sub>	F	α <sub>0</sub>	x <sub>1</sub>	F
B/X	0.0042 (0.65)	1.795 (14.2)	203.63	0.0106 (5.05)	1.134 (2.78)	7.78
τ	0.0292 (0.77)	0.1486 (1.45)	2.10	- 0.0531 (-4.34)	0.0359 (1.07)	1.16
σ	- 1.2001 (-17.7)	10.699 (17.9)	326.88	0.0941 (2.62)		14.59
TS	0.0993 (5.14)	- 15.512 (-13.7)	189.88	-0.0856 (-10.7)	3.840 (6.33)	40.32

Table 3					
Test	of errors: All curr	rencies."			

\*Numbers in the parentheses are t values.

The null hypothesis is  $\alpha_0 = 0$  and  $\alpha_1 = 0$ , or that the data cannot distinguish between prediction errors and systematic effects of  $Q_i$ . The results reported in table 3 indicate that the null hypothesis is rejected by both models for three of the four variables included. However, the systematic biases due to these variables are significantly lower for Model 2. In particular, the effects of time and term structure are much smaller in Model 2 than Model 1, indicating that the bulk of the errors related to these variables in Model 1 have been largely captured in Model 2 which utilizes yields with the same maturities as those of the option.

# 4. Concluding remarks

We have analyzed the effects of domestic and foreign yield curves on the value of foreign currency call options. In comparing the pricing performances of the currency option model which permits sloped yield curves and the one that does not, it is shown that the cost (pricing bias) of assuming constant interest rates is very high, and that incorporation of bond yields with maturities that match expiration dates of the option markedly improves the performance of the option pricing model.

Non-flat yield curves are related to the general interest rate uncertainty, but the focus of this paper has been on the former, not the latter. The latter can be examined more directly along the line of the model framework used in this paper regarding the pricing of foreign currency American options with stochastic interest rates, or by using another model such as Ramaswamy and Sundaresan (1985) modified to currency options.

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